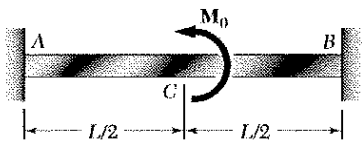
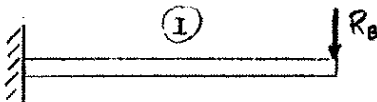


Problem 9.83

9.83 and 9.84 For the beam shown, determine the reaction at B.



Beam is second degree indeterminate. Choose R_B and M_B as redundant reactions.



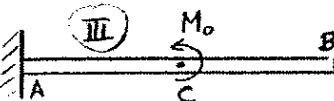
loading I. Case 1 of Appendix D.

$$(y_B)_I = -\frac{R_B L^3}{3EI}, \quad (\theta_B)_I = -\frac{R_B L^2}{2EI}$$



loading II. Case 3 of Appendix D.

$$(y_B)_{II} = \frac{M_B L^2}{2EI}, \quad (\theta_B)_{II} = \frac{M_B L}{EI}$$



loading III. Case 3 applied to portion AC.

$$(y_C)_{III} = \frac{M_0 (L/2)^2}{2EI} = \frac{M_0 L^2}{8EI}$$

$$(\theta_C)_{III} = \frac{M_0 (L/2)}{EI} = \frac{M_0 L}{2EI}$$

Portion CB remains straight.

$$(y_B)_{III} = (y_C)_{III} + \frac{L}{2}(\theta_C)_{III} = \frac{3}{8} \frac{M_0 L^2}{EI}$$

$$(\theta_B)_{III} = (\theta_C)_{III} = \frac{1}{2} \frac{M_0 L}{EI}$$

Superposition and constraint:

$$y_B = (y_B)_I + (y_B)_{II} + (y_B)_{III} = 0$$

$$-\frac{L^3}{3EI} R_B + \frac{L^2}{2EI} M_B + \frac{3}{8} \frac{M_0 L^2}{EI} = 0 \quad (1)$$

$$\theta_B = (\theta_B)_I + (\theta_B)_{II} + (\theta_B)_{III} = 0$$

$$-\frac{L^2}{2EI} R_B + \frac{L}{EI} M_B + \frac{1}{2} \frac{M_0 L}{EI} = 0 \quad (2)$$

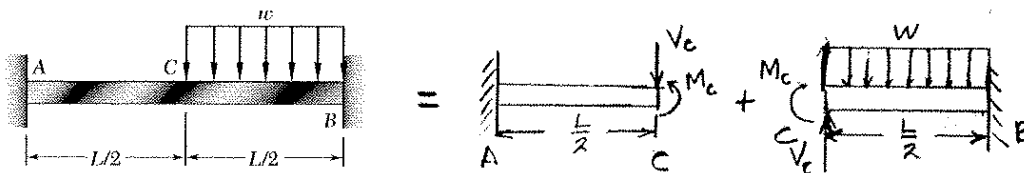
Solving (1) and (2) simultaneously,

$$R_B = \frac{3}{2} \frac{M_0}{L} \downarrow \quad \blacktriangleleft$$

$$M_B = \frac{1}{4} M_0 \curvearrowright \quad \blacktriangleleft$$

Problem 9.84

9.83 and 9.84 For the beam shown, determine the reaction at B.



Portion AC: Superposition of Cases 3 and 1 of Appendix D.

$$y_c = \frac{M_c(L/2)^2}{2EI} - \frac{V_c(L/2)^3}{3EI} = \frac{M_c L^2}{8EI} - \frac{V_c L^3}{24EI}$$

$$\theta_c = \frac{M_c(L/2)}{EI} - \frac{V_c(L/2)^2}{2EI} = \frac{M_c L}{2EI} - \frac{V_c L^2}{8EI}$$

Portion CB: Superposition of Cases 3, 1, and 2 of Appendix D.

$$y_c = \frac{M_c(L/2)^2}{2EI} + \frac{V_c(L/2)^3}{3EI} - \frac{w(L/2)^4}{8EI}$$

$$= \frac{M_c L^2}{8EI} + \frac{V_c L^3}{24EI} - \frac{wL^4}{128EI}$$

$$\theta_c = -\frac{M_c(L/2)}{EI} - \frac{V_c(L/2)^2}{2EI} + \frac{w(L/2)^3}{6EI}$$

$$= -\frac{M_c L}{2EI} - \frac{V_c L^2}{8EI} + \frac{wL^3}{48EI}$$

Matching expressions for y_c ,

$$\frac{M_c L^2}{8EI} - \frac{V_c L^3}{24EI} = \frac{M_c L^2}{8EI} + \frac{V_c L^3}{24EI} - \frac{wL^4}{128EI}$$

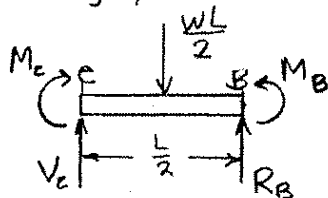
$$V_c = \frac{3}{32} wL$$

Matching expressions for θ_c ,

$$\frac{M_c L}{2EI} - \frac{V_c L^2}{8EI} = -\frac{M_c L}{2EI} - \frac{V_c L^2}{8EI} + \frac{wL^3}{48EI}$$

$$M_c = \frac{1}{48} wL^2$$

Using portion CB as a free body,



$$+\uparrow \sum F_y = 0: R_B + V_c - \frac{wL}{2} = 0$$

$$R_B = \frac{wL}{2} - \frac{3}{32} wL$$

$$R_B = \frac{13}{32} wL \quad \blacktriangleleft$$

$$+\circlearrowleft \sum M_B = 0: M_B - M_c - V_c \frac{L}{2} + \frac{wL}{2} \cdot \frac{L}{4} = 0$$

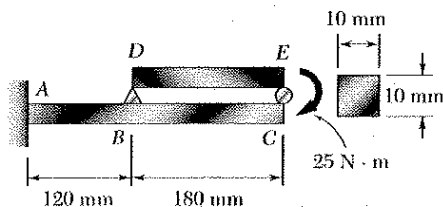
$$M_B = \frac{1}{48} wL^2 + \left(\frac{3}{32} wL\right) \left(\frac{L}{2}\right) - \frac{wL^2}{8}$$

$$M_B = -\frac{11}{192} wL^2$$

$$M_B = \frac{11}{192} wL^2 \quad \blacktriangleleft$$

Problem 9.85

9.85 Beam DE rests on the cantilever beam AC as shown. Knowing that a square rod of side 10 mm is used for each beam, determine the deflection at end C if the 25-N·m couple is applied (a) to end E of beam DE , (b) to end C of beam AC . Use $E = 200$ GPa.



$$E = 200 \times 10^9 \text{ Pa}$$

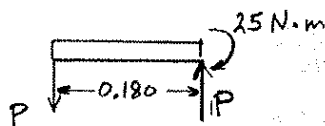
$$I = \frac{1}{12}(10)(10)^3 = 833.33 \text{ mm}^4 = 833.33 \times 10^{-12} \text{ m}^4$$

$$EI = 166.667 \text{ N}\cdot\text{m}^2$$

(a) Couple applied to beam DE .

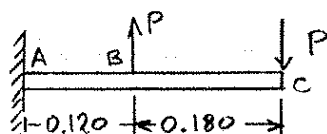
Free body DE . $+\circlearrowleft \Sigma M = 0$:

$$0.180 P - 25 = 0 \quad P = 138.889 \text{ N}$$



Loads on cantilever beam ABC are $P \uparrow$ at point B and $P \downarrow$ at point C as shown.

Due to $P \uparrow$ at point B .



Using portion AB and applying Case 1 of Appendix D,

$$(y_B)_1 = \frac{PL^3}{3EI} = \frac{(138.889)(0.120)^3}{(3)(166.667)} = 0.480 \times 10^{-3} \text{ m}$$

$$(\theta_B)_1 = \frac{PL^2}{2EI} = \frac{(138.889)(0.120)^2}{(2)(166.667)} = 6.00 \times 10^{-3}$$

$$(y_C)_1 = (y_B)_1 + L_{BC}(\theta_B)_1 = 0.480 \times 10^{-3} + (0.180)(6.00 \times 10^{-3}) = 1.56 \times 10^{-3} \text{ m}$$

Due to load $P \downarrow$ at point C . Case 1 of App. D applied to ABC .

$$(y_C)_2 = -\frac{PL^3}{3EI} = -\frac{(138.889)(0.120 + 0.180)^3}{(3)(166.667)} = -7.50 \times 10^{-3} \text{ m}$$

Total deflection at point C . $y_C = (y_C)_1 + (y_C)_2 = -5.94 \times 10^{-3} \text{ m}$

$$y_C = 5.94 \text{ mm} \downarrow$$

(b) Couple applied to beam AC .

Case 3 of Appendix D.

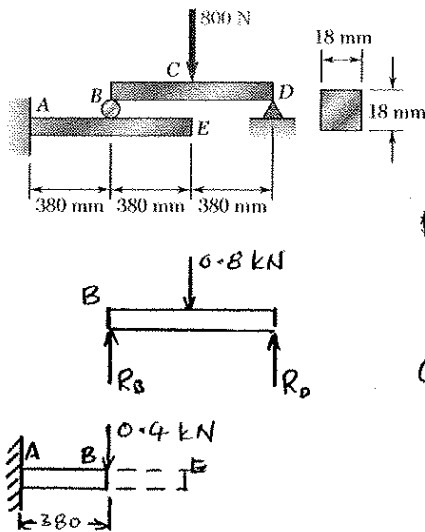
$$y_C = -\frac{ML^2}{2EI} = \frac{(25)(0.300)^2}{(2)(166.667)} = -6.75 \times 10^{-3} \text{ m}$$

$$y_C = 6.75 \text{ mm} \downarrow$$



Problem 9.86

9.86 Beam BD rests on the cantilever beam AE , as shown. Knowing that a square rod of side 18 mm is used for each beam, determine for the loading shown (a) the deflection at point C , (b) the deflection at point E . Use $E = 200 \text{ GPa}$.



$$I = \frac{1}{12}(18)(18)^3 = 8748 \text{ mm}^4$$

$$EI = (200 \times 10^6)(8748 \times 10^{-12}) = 1.75 \text{ kNm}^2$$

For free body BCD , $\sum M_D = 0$:

$$-760R_B + (380)(0.8) = 0 \quad R_B = 0.4 \text{ kN}$$

Cantilever ABE . Portion AB .

Case 1 of Appendix D.

$$y_B = -\frac{PL^3}{3EI} = -\frac{(0.4)(0.38)^3}{(3)(1.75)} = -4.181 \times 10^{-3} \text{ m}$$

$$\theta_B = -\frac{PL^2}{2EI} = -\frac{(0.4)(0.38)^2}{(2)(1.75)} = -16.5 \times 10^{-3} \text{ m}$$

Deflection at point C if point B does not move.

Apply Case 4 of Appendix D to beam BCD .

$$(y_C)_1 = -\frac{PL^3}{48EI} = -\frac{(0.8)(0.76)^3}{(48)(1.75)} = -4.181 \times 10^{-3} \text{ m}$$

Addition deflection at point C due to movement of point B .

$$(y_C)_2 = \frac{380}{760} y_B = -2.091 \times 10^{-3} \text{ m}$$

(a) Total deflection at point C . $y_C = (y_C)_1 + (y_C)_2 = -6.272 \times 10^{-3} \text{ m}$

$$y_C = 6.27 \text{ mm} \downarrow$$

(b) Deflection at point E . $y_E = y_B + L_{BE}\theta_B$

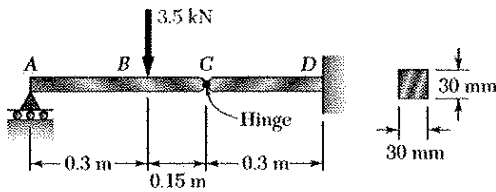
$$= -4.181 \times 10^{-3} + (0.38)(-16.5 \times 10^{-3})$$

$$= -10.451 \times 10^{-3} \text{ m}$$

$$y_E = 10.45 \text{ mm} \downarrow$$

Problem 9.87

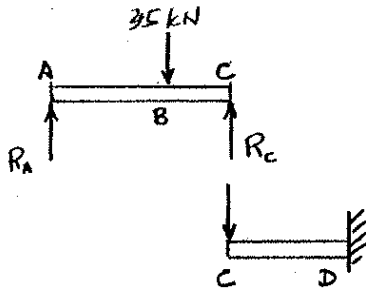
9.87 The two beams shown have the same cross section and are joined by a hinge at C. For the loading shown, determine (a) the slope at point A, (b) the deflection at point B. Use $E = 200 \text{ GPa}$.



Using free body ABC,

$$\sum M_A = 0: 0.45 R_C - (0.3)(35) = 0$$

$$R_C = 2.33 \text{ kN}$$



$$E = 200 \text{ GPa}$$

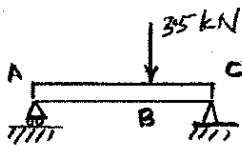
$$I = \frac{1}{12} b h^3 = \frac{1}{12} (30)(30)^3 = 67500 \text{ mm}^4$$

$$EI = (200 \times 10^9)(67500 \times 10^{-12}) = 13.5 \text{ kNm}^2$$

Using cantilever beam CD with load R_C ,

Case 1 of Appendix D.

$$y_C = -\frac{R_C L_{CD}^3}{3EI} = -\frac{(2.33)(0.3)^3}{(3)(13.5)} = -0.001555 \text{ m}$$



Calculation of θ_A' and y_B' assuming that point C does not move.

Case 5 of Appendix D

$$P = 35 \text{ kN} \quad L = 0.45 \text{ m}, \quad a = 0.3 \text{ m}, \quad b = 0.15 \text{ m}$$

$$\theta_A' = -\frac{Pb(L^2 - b^2)}{6EIL} = -\frac{(35)(0.15)(0.45^2 - 0.15^2)}{(6)(13.5)(0.45)} = -0.00259 \text{ rad}$$

$$y_B' = -\frac{Pb^2a^2}{3EIL} = -\frac{(35)(0.15)^2(0.3)^2}{(3)(13.5)(0.45)} = -0.000389 \text{ m}$$

Additional slope and deflection due to movement of point C.

$$\theta_A'' = \frac{y_C}{L_{AC}} = -\frac{0.001555}{0.45} = -0.003456 \text{ rad}$$

$$y_B'' = \frac{a}{L} y_C = -\frac{(0.3)(0.001555)}{0.45} = -0.0010367 \text{ m}$$

(a) Slope at A. $\theta_A = \theta_A' + \theta_A'' = -0.00259 - 0.003456$

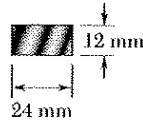
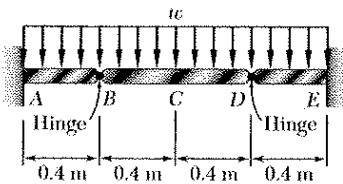
$$= -0.006046 \text{ rad} = 0.00605 \text{ rad} \quad \swarrow$$

(b) Deflection at B. $y_B = y_B' + y_B'' = -0.000389 - 0.0010367$

$$= -1.4257 \times 10^{-3} \text{ m} = 1.43 \text{ mm} \quad \downarrow$$

Problem 9.88

9.88 A central beam BD is joined at hinges to two cantilever beams AB and DE . All beams have the cross section shown. For the loading shown, determine the largest w so that the deflection at C does not exceed 3 mm. Use $E = 200$ GPa.



Let $a = 0.4$ m

Cantilever beams AB and CD .

Cases 1 and 2 of Appendix D; $y_c = -\frac{(wa)a^3}{3EI} - \frac{wa^4}{8EI} = -\frac{11}{24} \frac{wa^4}{EI}$

Beam BCD , with $L = 0.8$ m, assuming that points B and D do not move.

Case 6 of Appendix D.

$$y_c' = -\frac{5wL^4}{384EI}$$

Additional deflection due to movement of points B and D .

$$y_c'' = y_B = y_D = -\frac{11}{24} \frac{wa^4}{EI}$$

Total deflection at C .

$$y_c = y_c' + y_c''$$

$$y_c = -\frac{w}{EI} \left\{ \frac{5L^4}{384} + \frac{11a^4}{24} \right\}$$

Data: $E = 200 \times 10^9$ Pa, $I = \frac{1}{12}(24)(12)^3 = 3.456 \times 10^{-3} \text{ mm}^4 = 3.456 \times 10^{-9} \text{ m}^4$
 $EI = (200 \times 10^9)(3.456 \times 10^{-9}) = 691.2 \text{ N} \cdot \text{m}^2$

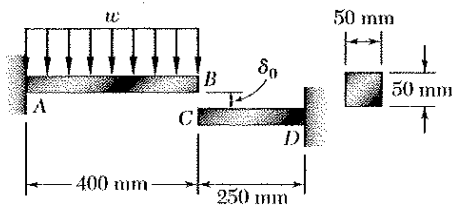
$$y_c = -3 \times 10^{-3} \text{ m}$$

$$-3 \times 10^{-3} = -\frac{w}{691.2} \left\{ \frac{(5)(0.8)^4}{384} + \frac{(11)(0.4)^4}{24} \right\} = -24.69 \times 10^{-6} w$$

$$w = 121.5 \text{ N/m} \rightarrow$$

Problem 9.89

9.89 Before the uniformly distributed load w is applied, a gap, $\delta_0 = 1.2$ mm, exists between the ends of the cantilever bars AB and CD . Knowing that $E = 105$ GPa and $w = 30$ kN/m, determine (a) the reaction at A , (b) the reaction at D .



$$I = \frac{1}{12}(50)(50)^3 = 520.833 \times 10^3 \text{ mm}^4 = 520.833 \times 10^{-7} \text{ m}^4$$

$$EI = (105 \times 10^9)(520.833 \times 10^{-7}) = 54.6875 \times 10^3 \text{ N}\cdot\text{m}^2$$

$$= 54.6875 \text{ kN}\cdot\text{m}^2$$

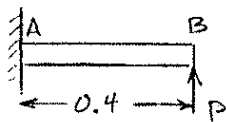
Units: Forces in kN. Lengths in meters.

Compute deflection at B due to w . Case 8 of Appendix D.

$$(y_B)_1 = -\frac{wL^4}{8EI} = -\frac{(30)(0.400)^4}{(8)(54.6875)} = -1.75543 \times 10^{-3} = -1.7553 \text{ mm}$$

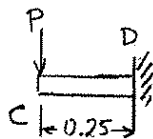
The displacement is more than δ_0 , the gap closes.

Let P be the contact force between points B and C .



Compute deflection of B due to P . Use Case 1 of Appendix D.

$$(y_B)_2 = \frac{PL^3}{3EI} = \frac{P(0.4)^3}{(3)(54.6875)} = 390.095 \times 10^{-6} P \text{ m}$$



Compute deflection of C due to P .

$$y_C = -\frac{PL^3}{3EI} = -\frac{P(0.25)^3}{(3)(54.6875)} = -95.238 \times 10^{-6} P \text{ m}$$

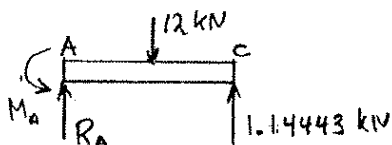
Displacement condition. $y_B + \delta_0 = y_C$

Using superposition, $(y_B)_1 + (y_B)_2 - \delta_0 = y_C$

$$-1.75543 \times 10^{-3} + 390.095 \times 10^{-6} P + 1.2 \times 10^{-3} = -95.238 \times 10^{-6} P$$

$$485.333 \times 10^{-6} P = 0.55543 \times 10^{-3} \quad P = 1.14443 \text{ kN.}$$

(a) Reaction at A.



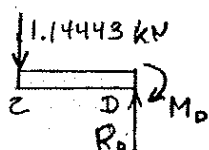
$$+\uparrow \Sigma F_y = 0: R_A - 12 + 1.14443 = 0$$

$$R_A = 10.86 \text{ kN} \uparrow$$

$$+\circlearrowleft \Sigma M_A = 0: M_A - (0.2)(12) + (0.4)(1.14443) = 0$$

$$M_A = 1.942 \text{ kN}\cdot\text{m} \circlearrowleft$$

(b) Reaction at D.



$$+\uparrow \Sigma F_y = 0: R_D - 1.14443 = 0$$

$$R_D = 1.144 \text{ kN} \uparrow$$

$$+\circlearrowleft \Sigma M_D = 0: -M_D + (0.25)(1.14443) = 0$$

$$M_D = 0.286 \text{ kN}\cdot\text{m} \circlearrowleft$$